Applications of Integrals

Introduction

Area Under Simple Curve

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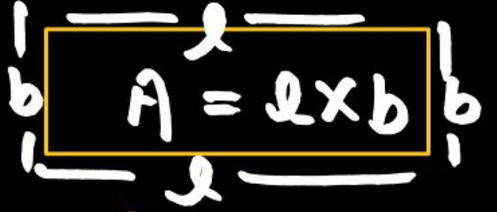
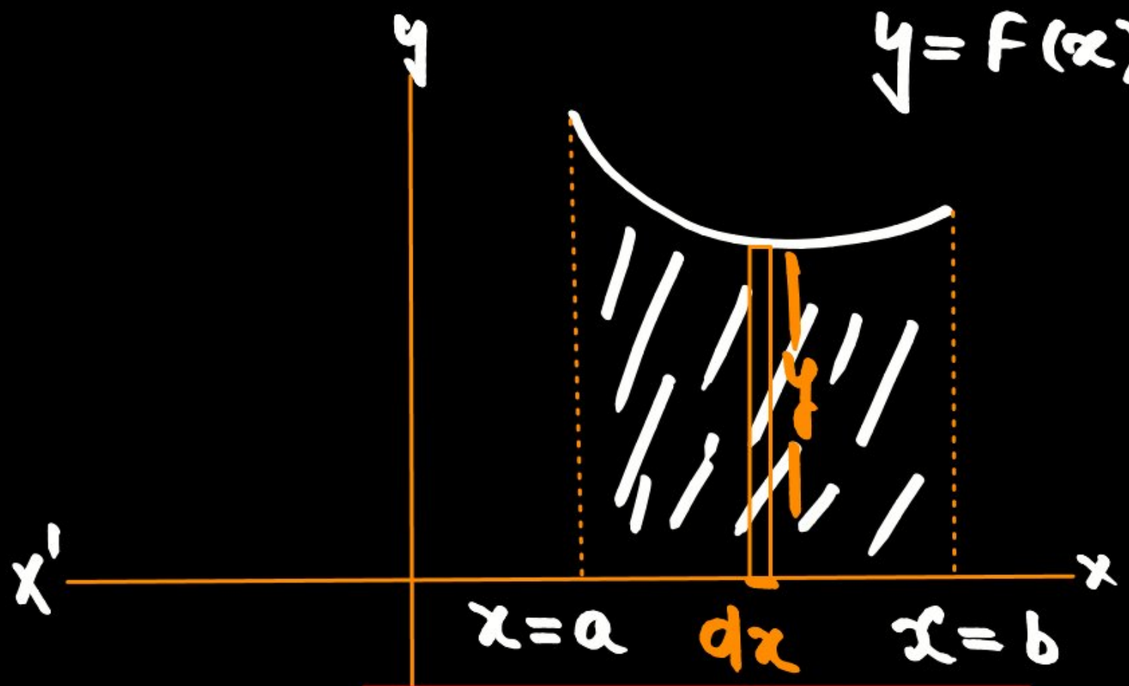
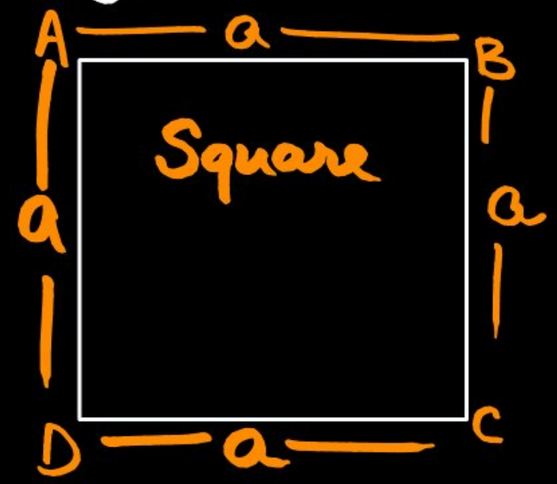
Introduction

Integration- Addition



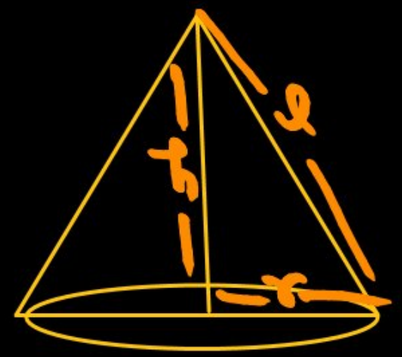




Area = a x a

$$y' \text{Area} = \int_{x=a}^{x=b} y\,dx$$

y = is terms of x



V. cone = $\frac{1}{3}\pi r^2 h$

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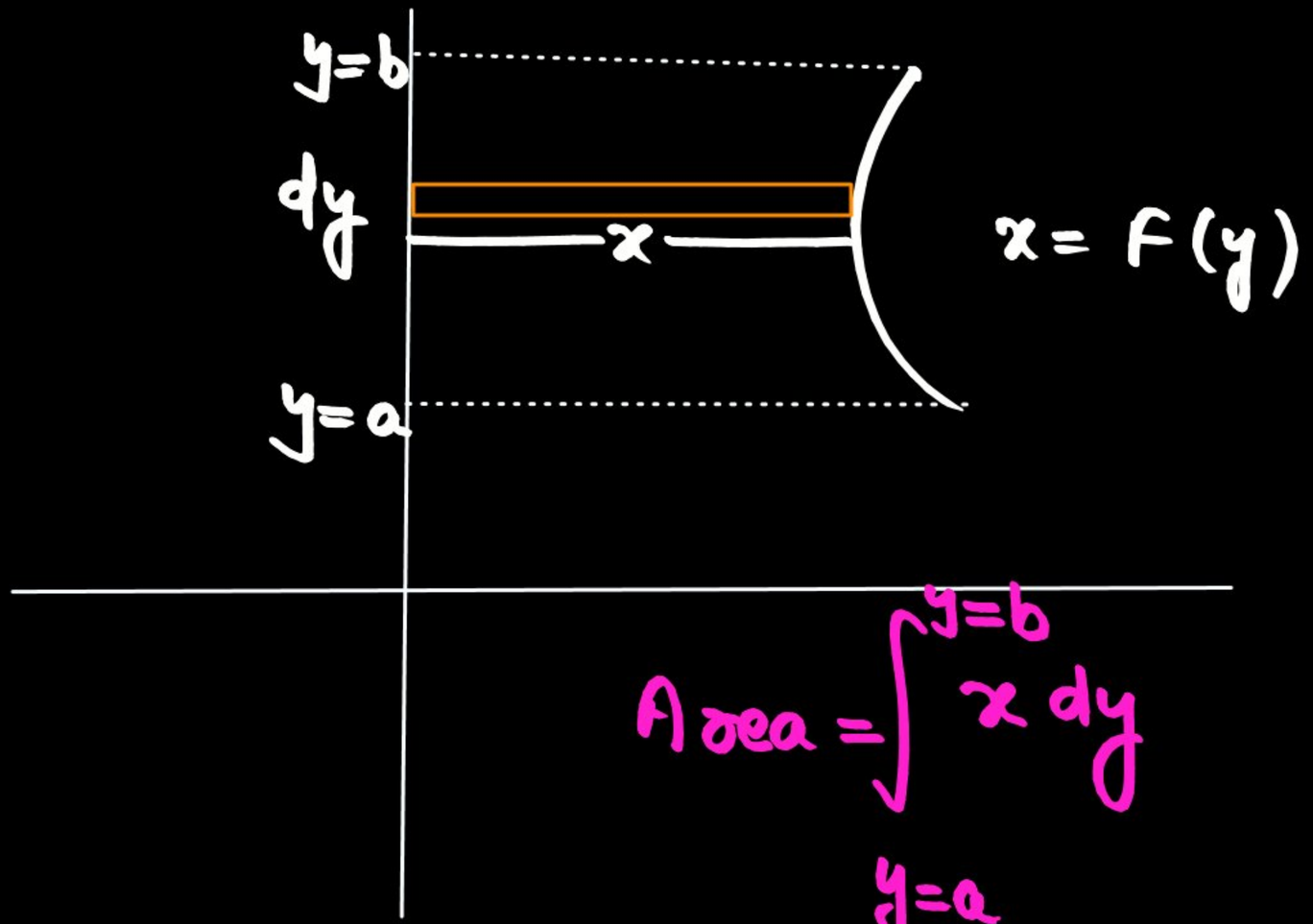

$$\text{Area} = \int_{y=a}^{y=b} x \, dy$$

$x \rightarrow$ in terms of y

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Area Under Simple Curve

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Formula

i) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

ii) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

iii) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

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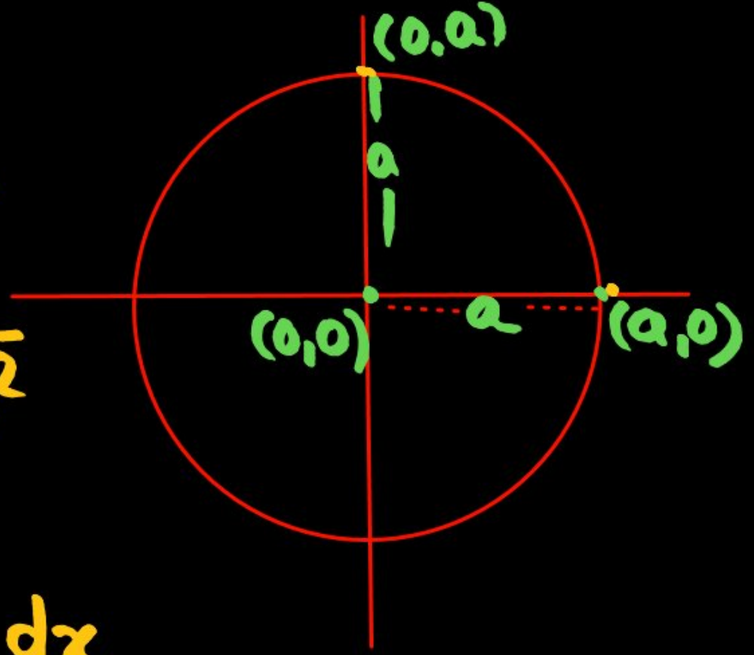
Q-1 Find the area enclosed by the circle $x^2 + y^2 = a^2$

$$x^2 + y^2 = a^2$$

$$y^2 = a^2 - x^2$$

$$y = \sqrt{a^2 - x^2}$$



$$\text{Area} = 4\int_0^a y\,dx$$

$$= 4\int_0^a \sqrt{a^2 - x^2}\,dx = 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$(x-h)^2 + (y-k)^2 = r^2$$

Centre (h, k)

Radius $= r$

$(0,0)$

$$x^2 + y^2 = r^2$$

Area can not be negative

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$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$4\left[\frac{a}{2}\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right]$$

$$4\left[0 + \frac{a^2}{2}\sin^{-1}\sin\frac{\pi}{2}\right]$$

$$4\left[\frac{a^2}{2}\sin^{-1}\frac{\pi}{2}\right]$$

$$4\left[\frac{a^2}{2}\times\frac{\pi}{2}\right] = \frac{\pi a^2}{4}\times 4 = \pi a^2 \text{ Sq. Unit}$$

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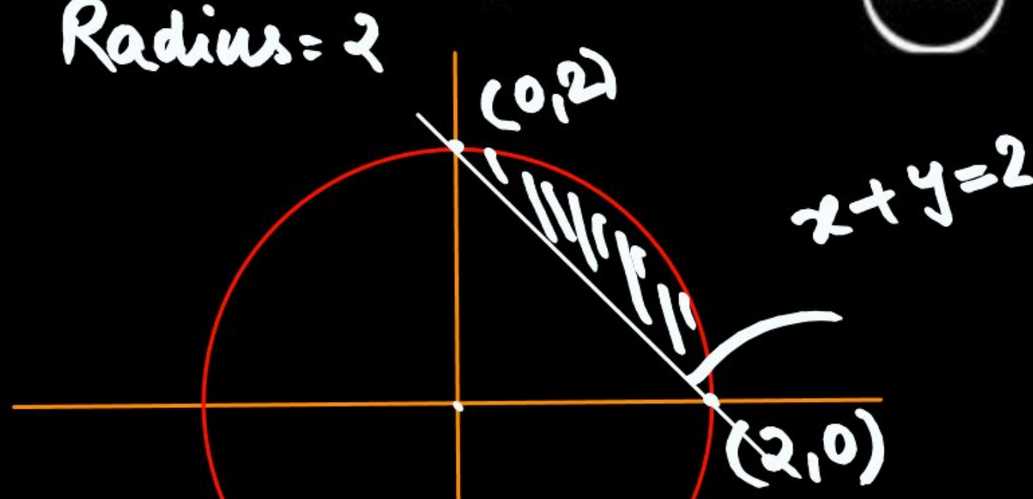
Find the smallest area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ $\Rightarrow y = 2 - x$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{(2)^2 - x^2}$

$x = 0$	$y = 0$
$y = 2$	$x = 2$
$(0,2)$	$(2,0)$



Area of Smallest enclosed fig $= \int_0^2 \sqrt{(2)^2 - x^2}\,dx - \int_0^2 (2-x)\,dx$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$

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$$= \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$

$$= \left[0 + 2\sin^{-1}\sin\frac{\pi}{2} - 0\right] - \left[4 - 2\right]$$

$$= \left[\cancel{2} \times \frac{\pi}{\cancel{2}}\right] - 2$$

$$= (\pi - 2) \text{ Sq. Unit}$$

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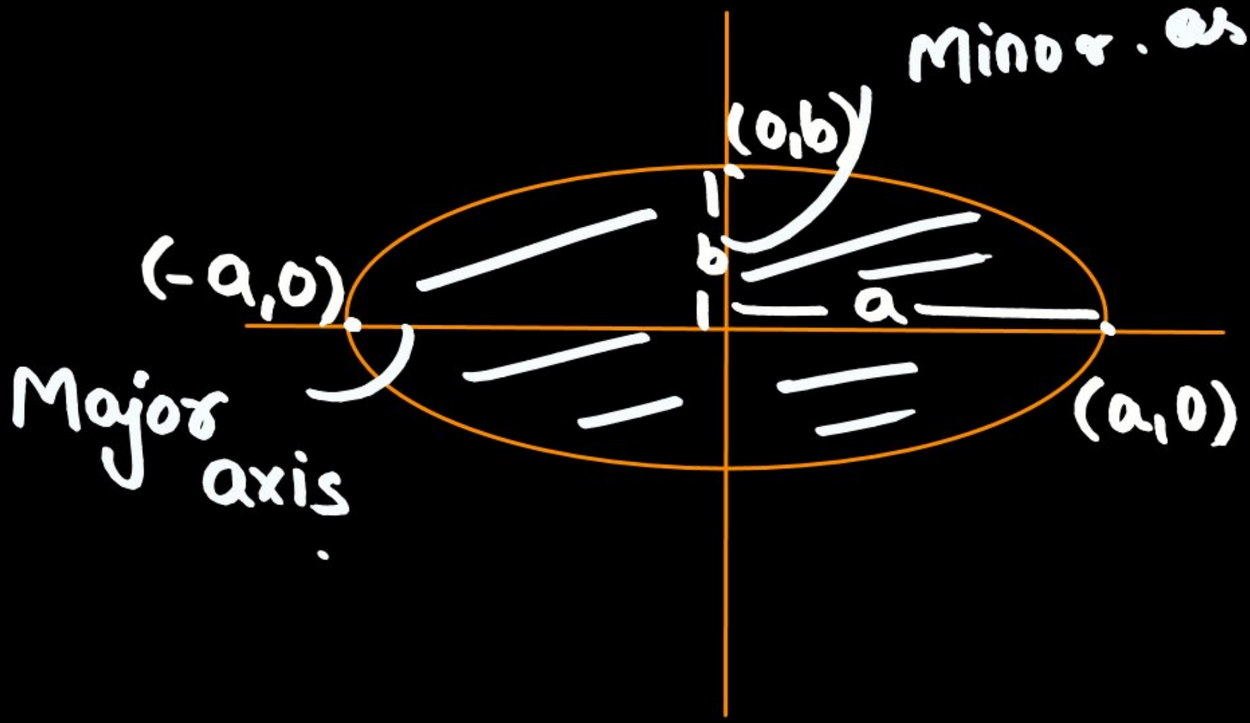
Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$y^2=\frac{b^2}{a^2}(a^2-x^2)$$

$$\boxed{y=\frac{b}{a}\sqrt{a^2-x^2}}$$



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$$\text{Area} = 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{4b}{a}\left[0 + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right]$$

$$= \frac{4b}{a}\left[\frac{a^2}{2}\sin^{-1}\sin\frac{\pi}{2}\right] = \frac{4b}{a}\left[\frac{a^2}{2}\times\frac{\pi}{2}\right] = \pi ab \text{ Sq. Unit}$$

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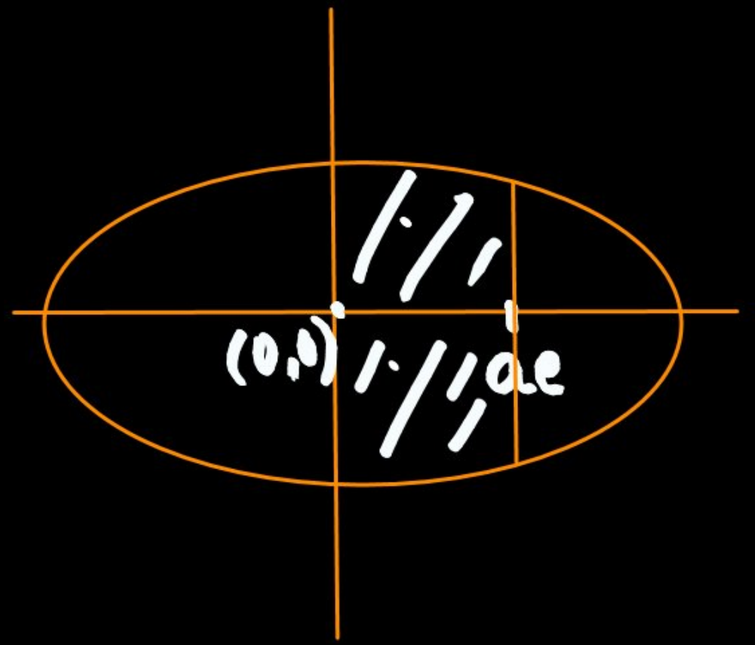
Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the ordinates $x=0$ and $x=ae$ where $b^2=a^2(1-e^2)$ and 0<e<1

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$y^2=\frac{b^2}{a^2}\sqrt{a^2-x^2}$$

$$\boxed{y=\frac{b}{a}\sqrt{a^2-x^2}}$$



$$\text{Area}=2\int_0^{ae} y\,dx=2\int_0^{ae}\frac{b}{a}\sqrt{a^2-x^2}\,dx$$

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$$= \frac{2b}{a} \int_0^{ae} \sqrt{a^2 - x^2}\, dx$$

$$= \frac{2b}{a} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^{ae}$$

$$= \frac{2b}{a} \left[\frac{ae}{2} \sqrt{a^2 - a^2e^2} + \frac{a^2}{2} \sin^{-1} \frac{ae}{a} \right] - 0$$

$$= \frac{2b}{a} \left[\frac{ae}{2} \times a \sqrt{1 - e^2} + \frac{a^2}{2} \sin^{-1} e \right]$$

$$= \frac{2ba^2}{2a} \left[e\sqrt{1 - e^2} + \sin^{-1} e \right] = ab \cdot \left[e\sqrt{1 - e^2} + \sin^{-1} e \right]$$ Ans

Sq. Unit

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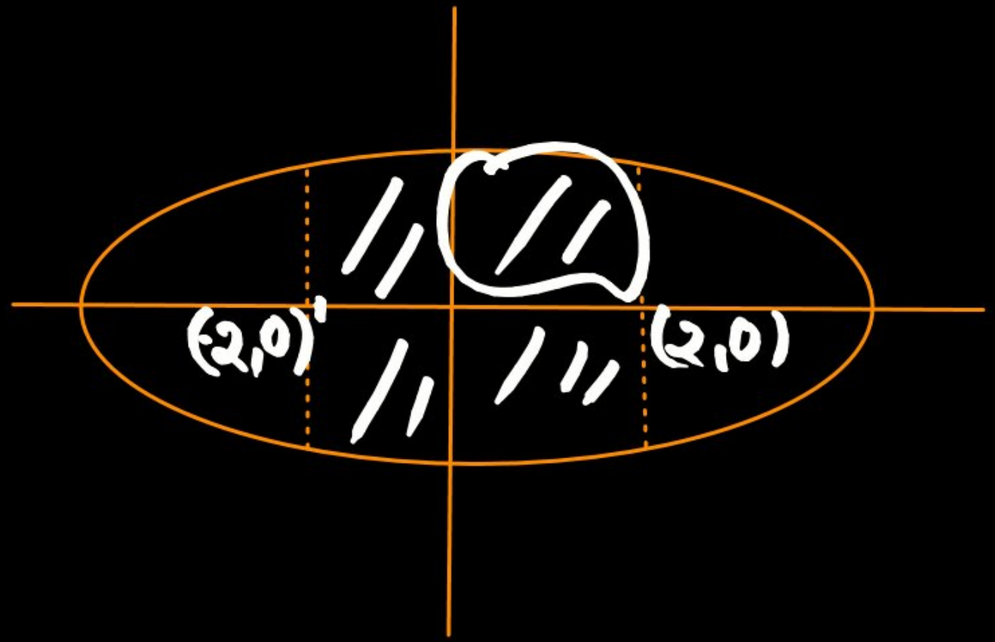
Find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$ included between the lines x=-2 and x=2

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$

$$\frac{y^2}{4}=1-\frac{x^2}{16}$$

$$y=\frac{4}{16}\sqrt{16-x^2}$$

$$\boxed{y=\frac{1}{2}\sqrt{16-x^2}}$$



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$$\text{Area} = \frac{1}{2}\int_0^2 \sqrt{16-x^2}\,dx$$

$$= \frac{1}{2}\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^2$$

$$= \frac{1}{2}\left[\frac{2}{2}\sqrt{16-4} + 4\sin^{-1}\frac{2}{4}\right]$$

$$= \frac{1}{2}\left[\sqrt{12} + 4\sin^{-1}\sin^{-1}\frac{\pi}{6}\right]$$

$$= \frac{1}{2}\left[2\sqrt{3} + \overset{2}{\cancel{4}} \times \frac{\pi}{\cancel{6}_3}\right] = \left[\frac{2}{2}\sqrt{3} + 2\frac{\pi}{3}\right] \text{Sq.}$$

If enclosed

Complete 4 part

$$= 4\left[\sqrt{3} + 2\frac{\pi}{3}\right]$$

$$= 4\sqrt{3} + 8\frac{\pi}{3} \text{ Ans}$$

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Find the area of the region bounded by $y^2 = 4x, x = 1 \; and \; x = 4$ and the x-axis in the first quadrant

$$A = \int_1^4 y \, dx$$

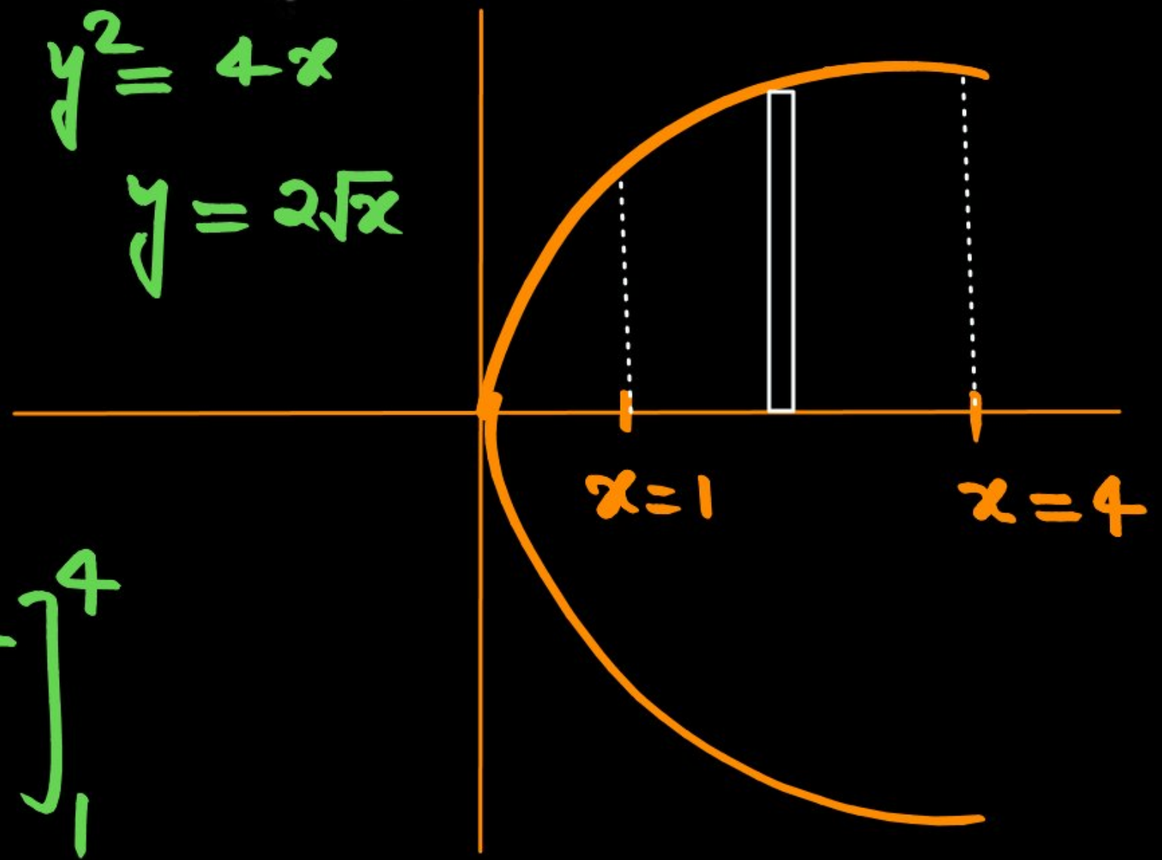
$$= \int_1^4 2\sqrt{x} \, dx$$

$$= 2\int_1^4 \sqrt{x} \, dx = 2\left[\frac{2x^{3/2}}{3}\right]_1^4$$

$$= \frac{4}{3}\left[(4)^{3/2} - 1\right] = \frac{4}{3}\left[0 - 1\right] = \frac{4}{3} \times 7 = \frac{28}{3} \text{ Sq. Unit}$$

$y^2 = 4x$

$y = 2\sqrt{x}$



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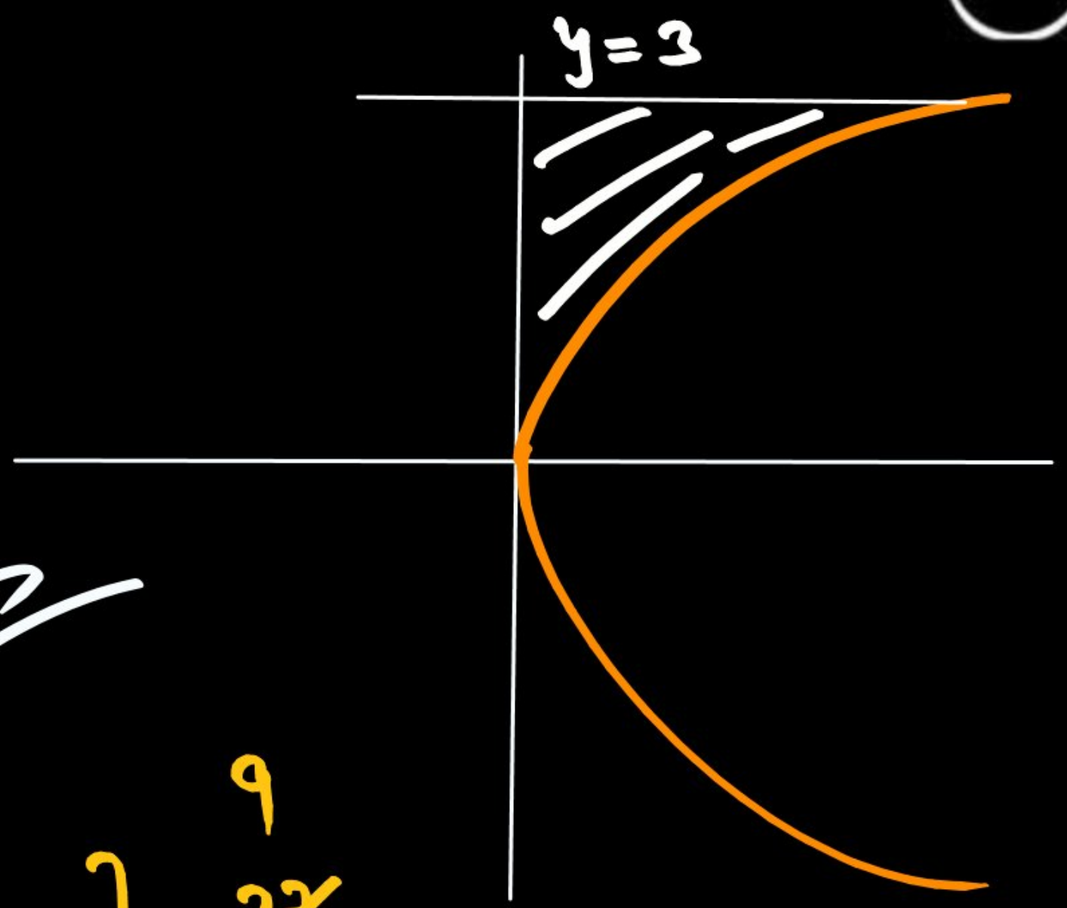
Find the area of the region bounded the curve $y^2 = 4x$, $y-axis\ and$ the line y=3



$y^2 = 4x$

$x = \frac{y^2}{4}$

$$\text{Area} = \int_0^3 \frac{y^2}{4}\,dy$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{4}\left[\frac{(3)^3}{3} - 0\right] = \frac{9}{4}$$

$= 9/4$ Sq. Unit

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